Mécaflu chap. 0 : Statique des fluides

1.1. $f(t+dt) = f(t) + \frac{df}{dt}(t) \times dt$ Taylor 1er ordre

au voisinage de t

1.2. $\overrightarrow{dOM} = \vec{v}\,dt$ $\vec{v} = \dot{x}\,\vec{e_x} + \dot{y}\,\vec{e_y} + \dot{z}\,\vec{e_z}$

$$= \left| \begin{matrix} \dot{x}\,dt \\ \dot{y}\,dt \\ \dot{z}\,dt \end{matrix} \right. = \left| \begin{matrix} dx \\ dy \\ dz \end{matrix} \right.$$

1.3. $T(M)$

$T(\vec{r})$ $\vec{r} = \overrightarrow{OM}$ avec O origine d'un repère.

$T(x,y,z)$

$\left(\frac{\partial T}{\partial x}\right)_{y,z}$ $\frac{\partial T}{\partial x}$

$f(x,y) = 2x^2y - 4\ln(x)e^y$

$\frac{\partial f}{\partial x} = 4yx - \frac{4e^y}{x}$

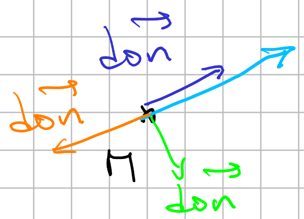
$$\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left(\frac{\partial T}{\partial x}\right) = \frac{\partial^2 T}{\partial x \partial y}$$

1.4. $T(x,y,z)$

$$dT = \frac{\partial T}{\partial x}dx + \frac{\partial T}{\partial y}dy + \frac{\partial T}{\partial z}dz$$

$$\boxed{dT = \overrightarrow{grad}(T) \cdot \overrightarrow{dOM}}$$

$$\left| \begin{matrix} \frac{\partial T}{\partial x} \\ \frac{\partial T}{\partial y} \\ \frac{\partial T}{\partial z} \end{matrix} \right. \qquad \left| \begin{matrix} dx \\ dy \\ dz \end{matrix} \right.$$



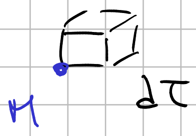
$\overrightarrow{grad}(T)$

$dT > 0$ $T \uparrow$

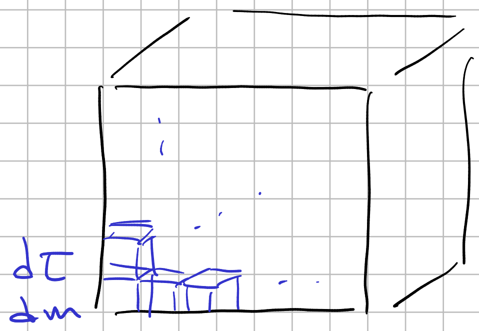
$dT < 0$ $T \downarrow$

$dT = 0$ T C^{te}

2.2.



$$\underset{kg}{dm} \overset{déf}{=} \underbrace{\rho(M)}_{kg\,m^{-3}} \underset{m^3}{d\tau}$$

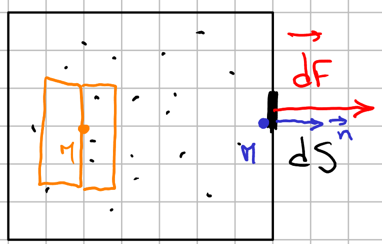


macro vol. V, masse m_{tot}

Extensivité : $m_{tot} = \iiint_{vol.\,V} dm$

$$\boxed{m_{tot} = \iiint_{M \in vol\,V} \rho(M)\, d\tau}$$

3.1.

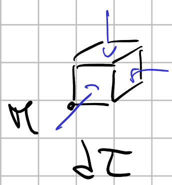


$$\underset{N}{\vec{dF}} \overset{déf}{\underset{(-)}{\underset{(+)}{=}}} \overset{>0}{P(M)}\, dS\, \underset{(\ldots)}{\vec{n}}$$

$$= (\pm)\, \underbrace{P(M)}_{\substack{N.m^{-2} \\ Pa}}\ \underset{m^2}{\vec{dS}}$$

$\hookrightarrow dS\,\vec{n}$

3.2.



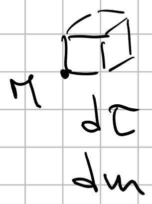
$$\vec{dF}_{pes} = dm\, \vec{g} = \underbrace{\left(\rho(M)\,\vec{g}\right)}_{\overset{déf}{=}\ \vec{f}_{pes}} \cdot d\tau$$

$$\boxed{\vec{f}_{pes} = \rho\,\vec{g}}$$

$N\ m^{-3}$

$$\vec{dF}_{\substack{pressio \\ tot}} = \underbrace{-\vec{grad}(P)}_{\overset{déf}{=}\ \vec{f}_{pression}} \times d\tau$$

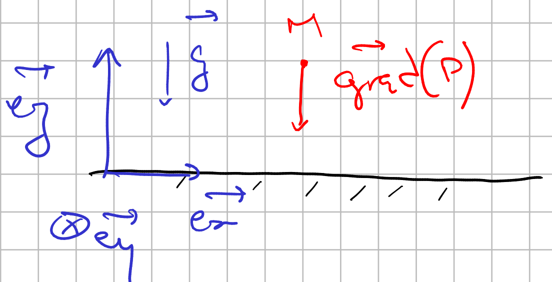
4.1. PFD dans réf^{el} terrestre galiléen :



$$\underbrace{dm \times \vec{a}}_{\substack{\vec{0} \\ \text{car statique}}} = \vec{dF}_{pes} + \vec{dF}_{\substack{press \\ tot}}$$

$$= (\rho \vec{g})\,\cancel{d\tau} + \left(-\vec{grad}(P)\right)\cancel{d\tau}$$

$$\boxed{-\vec{grad}(P) + \rho \vec{g} = \vec{0}} \qquad N\,m^{-3}$$



$\vec{grad}(P) = \rho \vec{g}$

$(\vec{e_x})$: $-\dfrac{\partial P}{\partial x} + 0 = 0$

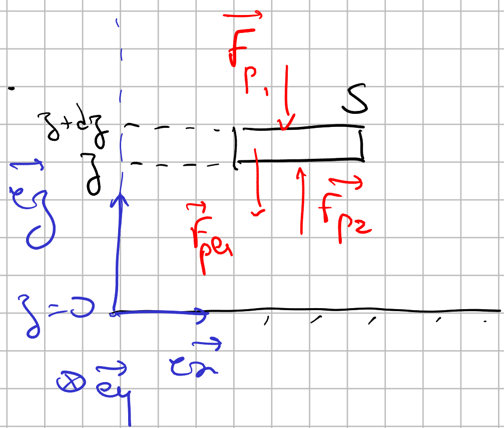
$(\vec{e_y})$: $-\dfrac{\partial P}{\partial y} + 0 = 0$

$(\vec{e_z})$ $-\dfrac{\partial P}{\partial z} - \rho g = 0$

$P(z)$

$$\boxed{\frac{dP}{dz} + \rho g = 0}$$

4.2.



$d\tau = S \times dz$

PFD syst : $\vec{0} = \vec{F}_{pes} + \vec{F}_{p_1} + \vec{F}_{p_2}$

$\vec{F}_{p_1} = -P(z+dz) \times S\,\vec{e_z}$

$\vec{F}_{p_2} = P(z) \times S\,\vec{e_z}$

$\vec{F}_{pes} = dm\,\vec{g} = -\rho(z)\,d\tau\,g\,\vec{e_z}$

$(\vec{e_z})$: $\underbrace{\left[P(z) - P(z+dz)\right]}_{-\frac{dP}{dz}(z)\,\cancel{dz}}\cancel{S} - \rho(z)\,g\,\cancel{S}\,\cancel{dz} = 0$

$$\boxed{\frac{dP}{dz} + \rho g = 0}$$

5.2. $\dfrac{dP}{dz} + \rho g = 0$ $\quad P(z) = -\rho g z + A$

$\vec{e_z}$ ↑ $z=0$ sol

$P(z=0) = P_0 \Rightarrow P_0 = 0 + A$

$$\boxed{P(z) = P_0 - g\rho z}$$ évol° affine.

haut

$h > 0$

bas

$$\boxed{P_{bas} = P_{haut} + \rho g h}$$

6.1. $\boxed{M_{air}} = 0{,}8\, M_{N_2} + 0{,}2\, M_{O_2}$ $\qquad Z_N = 7 \quad A_N = 14$

$= 0{,}8 \times 28 + 0{,}2 \times 32$ $\qquad M_{N_2} = 28\ g\,mol^{-1}$

$= 28{,}8\ g\,mol^{-1} = \boxed{29\ g\,mol^{-1}}$

$PV = nRT_0 \qquad PV = \dfrac{m}{M_{air}} RT_0 \qquad \boxed{\rho = \dfrac{P\,M_{air}}{RT_0}}$

6.2. $\vec{e_z}$ ↑ $z=0$

$\dfrac{dP}{dz} + \rho(z)\, g = 0$

$$\boxed{\frac{dP}{dz} + \frac{M_{air} g}{RT_0} P = 0}$$

$P(z) = A\, e^{-\frac{M_{air} g}{RT_0} z} \qquad P(z=0) = P_0$

$$\boxed{P(z) = P_0\, e^{-\frac{M_{air} g}{RT_0} z}}$$

→ " $e^{-\frac{t}{RC}} \leftrightarrow e^{-t/\tau}$, $\tau = RC$ " $\quad \tau$ = tps χ^{ic}

Ici e^{-z/H_c} avec $\boxed{H_c \overset{déf}{=} \dfrac{RT_0}{M_{air} g}}$

AN^{ic} : $\boxed{H_c} = \dfrac{8 \times 300}{30\, 10^{-3} \times 10}$

$= 8\, 10^3\ m = \boxed{8\ km}$