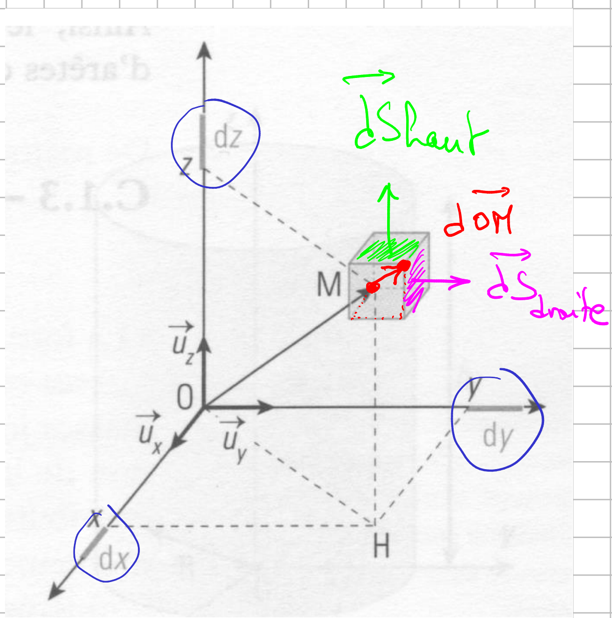
Annexe systèmes de coordonnées

$d\overrightarrow{OM} \stackrel{déf}{=} \overrightarrow{OM}_{final} - \overrightarrow{OM}_{initial}$



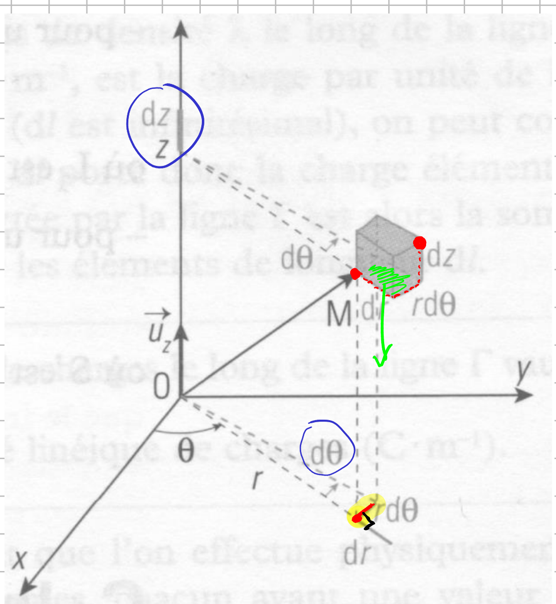
$\overrightarrow{dS} = dS\,\vec{n}$

$$\boxed{d\tau = dx\,dy\,dz}$$

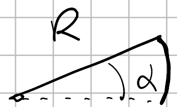
$\overrightarrow{dS}_{haut} = dx\,dy\,\overrightarrow{u_z}$

$\overrightarrow{dS}_{droite} = dx\,dz\,\overrightarrow{u_y}$

moyen mnémotechnique



Rappel : déf° angle α



$\ell = R \times \alpha$

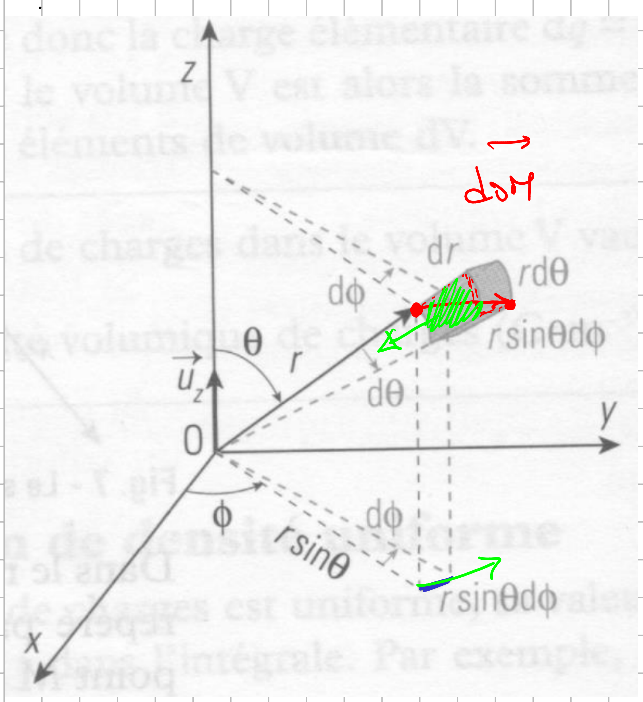
$\alpha \stackrel{déf}{=} \dfrac{\ell}{R}$

$$\boxed{d\overrightarrow{OM} = dr\,\overrightarrow{u_r} + r\,d\theta\,\overrightarrow{u_\theta} + dz\,\overrightarrow{u_z}}$$

$d\tau = dr \times r\,d\theta \times dz = r\,dr\,d\theta\,dz$

$\overrightarrow{dS}_{bas} = dr \times r\,d\theta\,(-\overrightarrow{u_z})$

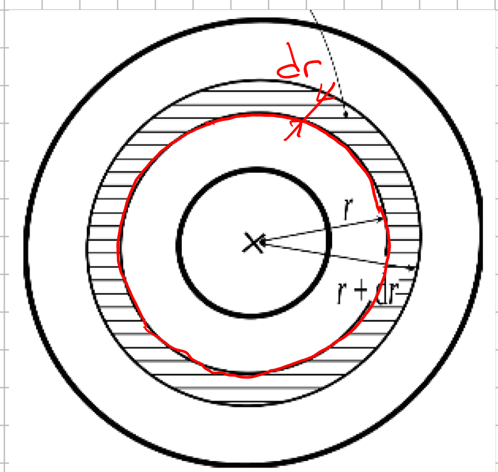
$= -r\,dr\,d\theta\,\overrightarrow{u_z}$

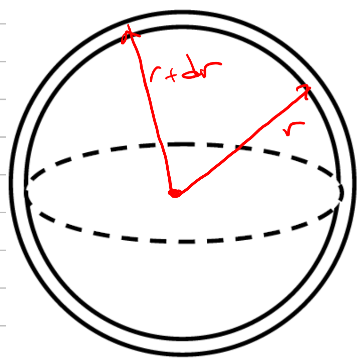


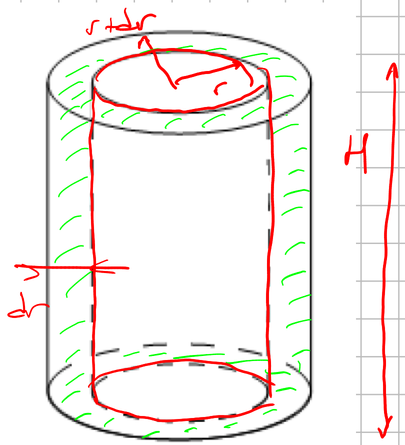
$$\boxed{\overrightarrow{dOM} = dr\,\overrightarrow{u_r} + r\,d\theta\,\overrightarrow{u_\theta} + r\sin\theta\,d\phi\,\overrightarrow{u_\phi}}$$

$$d\tau = dr \times r\,d\theta \times r\sin\theta\,d\phi$$

$$\overrightarrow{dS} = dr \times r\,d\theta\,\left(-\overrightarrow{u_\phi}\right) = -\,r\,dr\,d\theta\,\overrightarrow{u_\phi}$$







$$dS = 2\pi r \times dr$$

$$\begin{aligned} dS &= \pi (r+dr)^2 - \pi r^2 \\ &= \pi r^2 \left(1 + \frac{dr}{r}\right)^2 - \pi r^2 \\ &= \pi r^2 \left(1 + 2\frac{dr}{r}\right) - \pi r^2 \\ &= 2\pi r\,dr \end{aligned}$$

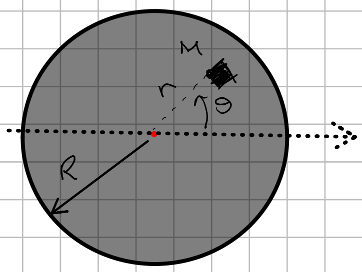
$$d\tau = 4\pi r^2 \times dr$$

$$d\tau = \frac{4}{3}\pi\left[(r+dr)^3 - r^3\right]$$

$$d\tau = 2\pi r H \times dr$$

$$\left(\text{Vol cyl} : \ \pi r^2 H\right)$$

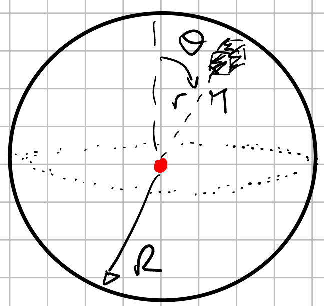
3.2. (Hors programme) Exp° vol. et surface via calcul intégral



$$dS = dr \times r d\theta$$

$$S_{tot} = \int_{r=0}^{R} \int_{\theta=0}^{2\pi} dS = \int_0^R \int_0^{2\pi} r dr \, d\theta$$

$$= \underbrace{\int_0^R r dr}_{\left[\frac{r^2}{2}\right]_0^R \to \frac{R^2}{2}} \times \underbrace{\int_0^{2\pi} d\theta}_{2\pi} = \pi R^2.$$



$$V_{tot} = \iiint_{sph} d\tau = \int_{r=0}^{R} \int_{\theta=0}^{\pi} \int_{\varphi=0}^{2\pi} dr \times r d\theta \times r \sin\theta \, d\varphi$$

$$= \underbrace{\int_0^R r^2 dr}_{\left[\frac{r^3}{3}\right]_0^R} \times \underbrace{\int_0^{\pi} \sin\theta \, d\theta}_{2} \times \underbrace{\int_0^{2\pi} d\varphi}_{2\pi} = \frac{4}{3}\pi R^3.$$