

# ANNEXE d'ANALYSE VECTORIELLE

## 1. Coordonnees cartesiennes

$$\text{Gradient : } \vec{\text{grad}} f = \frac{\partial f}{\partial x} \vec{u}_x + \frac{\partial f}{\partial y} \vec{u}_y + \frac{\partial f}{\partial z} \vec{u}_z$$

$$\text{Divergence : } \text{div} \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\text{Rotationnel : } \vec{\text{rot}} \vec{A} = \left( \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) \vec{u}_x + \left( \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) \vec{u}_y + \left( \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \vec{u}_z.$$

$$\text{Laplacien scalaire : } \Delta f = \text{div}(\vec{\text{grad}}(f)) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\text{Laplacien vectoriel : } \vec{\Delta} \vec{A} = (\Delta A_x) \vec{u}_x + (\Delta A_y) \vec{u}_y + (\Delta A_z) \vec{u}_z$$

## 2. Coordonnees cylindriques

$$\text{Gradient : } \vec{\text{grad}} f = \frac{\partial f}{\partial r} \vec{u}_r + \frac{1}{r} \frac{\partial f}{\partial \theta} \vec{u}_\theta + \frac{\partial f}{\partial z} \vec{u}_z$$

$$\text{Divergence : } \text{div} \vec{A} = \frac{1}{r} \frac{\partial}{\partial r} (r A_r) + \frac{1}{r} \frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z}$$

$$\text{Rotationnel : } \vec{\text{rot}} \vec{A} = \left( \frac{1}{r} \frac{\partial A_z}{\partial \theta} - \frac{\partial A_\theta}{\partial z} \right) \vec{u}_r + \left( \frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \right) \vec{u}_\theta + \left( \frac{1}{r} \frac{\partial (r A_\theta)}{\partial r} - \frac{1}{r} \frac{\partial A_r}{\partial \theta} \right) \vec{u}_z$$

$$\text{Laplacien scalaire : } \Delta f = \frac{\partial^2 f}{\partial r^2} + \frac{1}{r} \frac{\partial f}{\partial r} + \frac{1}{r^2} \frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 f}{\partial z^2} \quad \text{ou} \quad \Delta f = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial f}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 f}{\partial z^2}$$

## 3. Coordonnees spheriques

$$\vec{\text{grad}} f = \frac{\partial f}{\partial r} \vec{u}_r + \frac{1}{r} \frac{\partial f}{\partial \theta} \vec{u}_\theta + \frac{1}{r \sin \theta} \frac{\partial f}{\partial \varphi} \vec{u}_\varphi$$

$$\text{div} \vec{A} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 A_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta A_\theta) + \frac{1}{r \sin \theta} \frac{\partial A_\varphi}{\partial \varphi}$$

$$\vec{\text{rot}}(\vec{A}) = \frac{1}{r \sin \theta} \left( \frac{\partial}{\partial \theta} (\sin \theta A_\varphi) - \frac{\partial A_\theta}{\partial \varphi} \right) \vec{u}_r + \left( \frac{1}{r \sin \theta} \frac{\partial A_r}{\partial \varphi} - \frac{1}{r} \frac{\partial}{\partial r} (r A_\varphi) \right) \vec{u}_\theta + \frac{1}{r} \left( \frac{\partial}{\partial r} (r A_\theta) - \frac{\partial A_r}{\partial \theta} \right) \vec{u}_\varphi$$

$$\Delta f = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial f}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 f}{\partial \varphi^2}$$

## 4. Proprietés

$$\text{div}(\vec{\text{rot}}(\vec{A})) = 0$$

$$\vec{\text{rot}}(\vec{\text{grad}}(f)) = \vec{0}$$

$$\vec{\text{rot}}(\vec{\text{rot}}(\vec{A})) = \vec{\text{grad}}(\text{div}(\vec{A})) - \vec{\Delta} \vec{A}$$