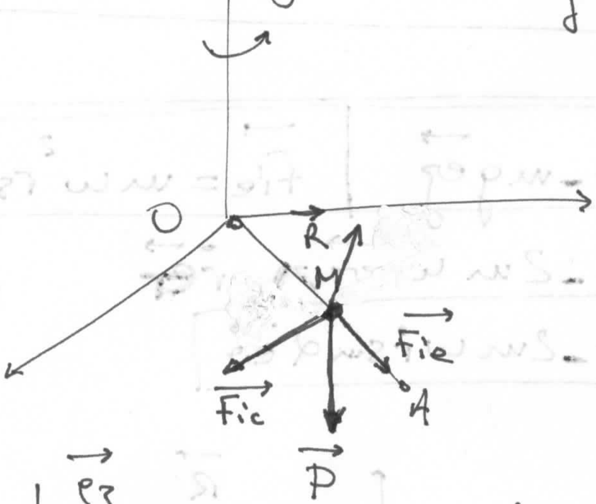
$\boxed{\text{Réf. non gal : CCPTSi 2007}}$ /27

1°/ [3]

$\vec{e_z}$ $\quad \vec{\Omega} = \omega \vec{e_z}$



→ poids | vertical descendant
$\vec{e_z}$

→ $\vec{Fie}$ | radiale centrifuge
$\vec{e_r}$

→ $\vec{Fic} = -2m\,\vec{\Omega} \wedge \vec{v}_{M/tige}$ selon $(-\vec{e_\theta})$

si $\vec{v}_{M/tige}$ selon $(+\vec{e_r})$

→ $\vec{R} \perp$ tige et compense $\vec{P}$ et $\vec{Fic}$

↳ selon $\vec{e_\theta}$ et $\vec{e_z}$ (ie $\perp \vec{e_r}$)

0,5

$\boxed{\vec{P} = -mg\,\vec{e_z}}$

$\boxed{\vec{Fie} = m\omega^2 r\,\vec{e_r}}$

1,5

$\boxed{\vec{Fic} = -2m\omega\vec{e_z} \wedge \dot{r}\vec{e_r} = -2m\omega\dot{r}\,\vec{e_\theta}}$

2°/ PFD dans réf. non gal tige : syst M [2]

$$m\ddot{r}\,\vec{e_r} = -mg\,\vec{e_z} + m\omega^2 r\,\vec{e_r} - 2m\omega\dot{r}\,\vec{e_\theta} + R_\theta\,\vec{e_\theta} + R_z\,\vec{e_z}$$

$(\vec{e_r})$: $\boxed{\ddot{r} - \omega^2 r = 0}$

3°/ [3] $x^2 - \omega^2 = 0 \qquad x = \pm\omega$

$\boxed{r(t) = Ae^{\omega t} + Be^{-\omega t}}$

C.i : $\left|\begin{array}{l} r(0) = r_0 \\ \dot{r}(0) = 0 \end{array}\right\} \quad \left|\begin{array}{l} A + B = r_0 \\ \omega(A - B) = 0 \end{array}\right. \quad \left|\begin{array}{l} A = B \\ A = \dfrac{r_0}{2} \end{array}\right.$

$\boxed{r(t) = \dfrac{r_0}{2}\left(e^{\omega t} + e^{-\omega t}\right)} \quad (= r_0\,ch(\omega t))$

4/ $\underline{\tau \text{ tq } r(\tau) = \ell}$: $\mathrm{ch}(\omega\tau) = \frac{\ell}{r_0}$ [2]

$$\boxed{\tau = \frac{1}{\omega}\,\mathrm{argch}\left(\frac{\ell}{r_0}\right)}$$

5/ $\vec{v}_f = \vec{v}(t=\tau) = \dot{r}(\tau)\,\vec{e_r}$ [5]

(+1) $\dot{r}(t) = r_0\omega\,\mathrm{sh}(\omega t)$

$$\vec{v}_f = r_0\omega\,\mathrm{sh}\left(\frac{\omega}{\omega}\,\mathrm{argch}\left(\frac{\ell}{r_0}\right)\right)\vec{e_r}$$

$$\boxed{\vec{v}_f = r_0\omega\,\mathrm{sh}\left[\mathrm{argch}\left(\frac{\ell}{r_0}\right)\right]\vec{e_r}}$$ (unités ok) 2

$\underline{Rq}$: $\mathrm{ch}^2 - \mathrm{sh}^2 = 1$

$$\mathrm{sh}\left[\mathrm{argch}\left(\frac{\ell}{r_0}\right)\right] = \sqrt{\left(\frac{\ell}{r_0}\right)^2 - 1}$$ (+1)

$\vec{v}_f{}' = \vec{v}_f + \underbrace{\vec{v}_e}_{\vec{v}_{P_l\,\text{sol}}}$] $\underline{\text{vitesse pt coïncident}}$ → mvt circulaire uniforme de rayon r, vit. angulaire ω

$\vec{OP} = \underbrace{r}_{\text{cte}}\,\vec{e_r}$

$\vec{v}_P = r\omega\,\vec{e_\theta}$

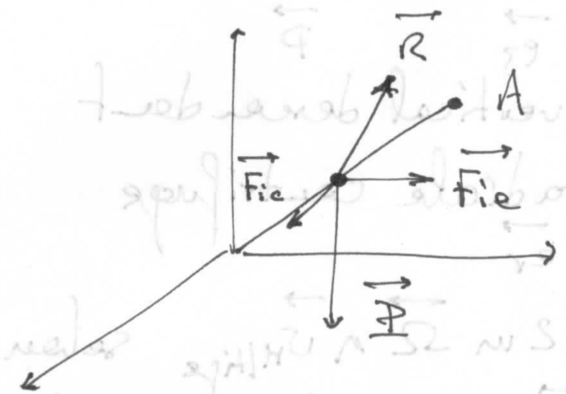
$$\boxed{\vec{v}_f{}' = r_0\omega\left[\mathrm{ch}(\omega\tau)\,\vec{e_\theta} + \mathrm{sh}(\omega\tau)\,\vec{e_r}\right]}$$

$$\boxed{\vec{v}_f{}' = r_0\omega\left[\frac{\ell}{r_0}\,\vec{e_\theta} + \sqrt{\left(\frac{\ell}{r_0}\right)^2 - 1}\;\vec{e_r}\right]}$$

6/ [3] $\boxed{\vec{P} = -mg\,\vec{e_z}}$ | $\vec{f_{ie}} = m\omega^2 r \sin\alpha\,\vec{e_r}$

$\vec{f_{ic}} = -2m\,\omega\,\vec{e_z} \wedge \dot{r}\,\vec{e_r}$ 2

$$\boxed{\vec{f_{ic}} = -2m\omega\dot{r}\sin\alpha\,\vec{e_\theta}}$$



7/ [1] $\underline{\text{PFD non gal sur M dans réf. tige}}$:

$$m\ddot{r}\,\vec{e_r} = -mg\,\vec{e_z} + m\omega^2 r\sin\alpha\,\vec{e_r} - 2m\omega\dot{r}\sin\alpha\,\vec{e_\theta} + \underbrace{\vec{R}}_{(\perp \vec{e_r})}$$

7°/ $(\vec{e_T})$: $m\ddot{r} = -mg\cos\alpha + m\omega^2\sin^2\alpha\, r$ $\boxed{2}$

$$\Rightarrow \boxed{\ddot{r} - \omega^2\sin^2\alpha\, r = -g\cos\alpha}$$

8°/ $\boxed{5}$ Sol° part : $\boxed{r_p = \dfrac{g\cos\alpha}{\omega^2\sin^2\alpha}}$ $(\alpha \neq 0)$

Sol° ESSM : $\boxed{r_{essm} = A e^{\omega\sin\alpha t} + B e^{-\omega\sin\alpha t}}$

Sol° gé : $\boxed{r(t) = \dfrac{g\cos\alpha}{\omega^2\sin^2\alpha} + A e + B e^{-}}$

C.i. sur sol° gé : $r(0) = r_0$ et $\dot{r}(0) = 0$

$$\left.\begin{array}{l} \dfrac{g\cos\alpha}{\omega^2\sin^2\alpha} + A + B = r_0 \\ A = B \end{array}\right\} \quad \boxed{A = \frac{r_0}{2} - \frac{g\cos\alpha}{2\omega^2\sin^2\alpha}}$$

$$\boxed{r(t) = \frac{g\cos\alpha}{\omega^2\sin^2\alpha} + \left[r_0 - \frac{g\cos\alpha}{\omega^2\sin^2\alpha}\right] \mathrm{ch}(\omega\sin\alpha t)}$$

9°/ $\boxed{3}$ Eq $\Leftrightarrow$ $\ddot{r} = 0 \Leftrightarrow$ $\boxed{r_{eq} = \dfrac{g\cos\alpha}{\omega^2\sin^2\alpha}}$

$r_{eq} \leqslant \ell \Leftrightarrow g\cos\alpha \leqslant \ell\omega^2\sin^2\alpha$

$$\Leftrightarrow \boxed{\omega \geqslant \underbrace{\sqrt{\frac{g\cos\alpha}{\ell\sin^2\alpha}}}_{\omega_0}}$$